Macroscopic simulation of widely scattered synchronized traffic states

1999 J. Phys. A: Math. Gen. 32 L17

(http://iopscience.iop.org/0305-4470/32/1/003)

J. Phys. A: Math. Gen. **32** (1999) L17–L23. Printed in the UK PII: S0305-4470(99)97222-0

LETTER TO THE EDITOR

Macroscopic simulation of widely scattered synchronized traffic states

Martin Treiber† and Dirk Helbing†‡

† II. Institute of Theoretical Physics, University of Stuttgart, Pfaffenwaldring 57, D-70550 Stuttgart, Germany

‡ Department of Biological Physics, Eötvös University, Budapest, Puskin u 5–7, H-1088, Hungary

Received 2 September 1998, in final form 11 November 1998

Abstract. Recently, a phase transition to synchronized congested traffic has been observed in empirical highway data (Kerner B S and Rehborn H 1997 *Phys. Rev. Lett.* **79** 4030). This hysteretic transition has been described by a non-local, gas-kinetic-based traffic model (Helbing D and Treiber M 1998 *Phys. Rev. Lett.* **81** 3042) that did not, however, display a wide scattering of synchronized states. Here, it is shown that the latter can be reproduced by a mixture of different vehicle types like cars and trucks. The simulation results are in good agreement with Dutch highway data.

Recent publications stressed the fact that, whereas in free traffic the observed flow–density diagram is well described by a unique one-dimensional flow–density relation, in congested traffic the empirical data points are distributed over a two-dimensional region [1, 2]. In order to account for this fact, Krauß [3] has recently proposed that driver behaviour is different in congested traffic than in free traffic. Lenz *et al* [4] and Schreckenberg [5], however, have suggested that the wide scattering of data is caused by an anticipation effect of drivers who not only react to the respective vehicle in front but also to the traffic dynamics further ahead.

In contrast to these 'microscopic' approaches which simulate the interactions of individual vehicles, macroscopic models describe the evolution of the macroscopic velocity $V(x,t) = \langle v_\alpha \rangle$ and the vehicle density $\rho(x,t) = \langle 1/s_\alpha \rangle$, which are local averages of the 'microscopic' velocities v_α of the vehicles α and their centre-to-centre distances s_α. All fluctuating quantities like individual velocity variations or distance distributions are eliminated. This means that, in deterministic macrosimulations, all self-organized structures (like stop-and-go waves or congested traffic) are smooth.

Therefore, some researchers believe that, while the wide scattering of the congested flow–density data may be reproduced by *microsopic* traffic models, *macroscopic* ones will fail for principal reasons. However, motivated by the fact that the scattering has been observed in aggregated rather than single-vehicle data, we are confident that a macroscopic simulation of this effect should be possible.

In the following, we will show that scattering can be explained by the fluctuations caused by a heterogeneous traffic population, which enter the macroscopic simulations *via* the boundary conditions. We will distinguish cars and trucks characterized by different sets of parameter values. These define two equilibrium flow–density relations of pure car traffic and pure truck traffic, respectively, which are close to each other at small vehicle densities, but considerably

0305-4470/99/010017+07$19.50

different in the congested density regime. For mixed traffic, we interpolate between both parameter sets and, hence, between both equilibrium relations, using a weighted average. The weights are extracted from real traffic data by determining the proportion of long vehicles (‘trucks’). This method allows us to simulate a uni-directional multi-lane freeway by an effective one-lane model for one car species with average, but varying parameter values. Our simulations are carried out with empirically measured boundary conditions, and the results are quite realistic.

For the simulations, we use the macroscopic, gas-kinetic-based traffic (GKT) model [6,7], which shows a realistic instability diagram and the characteristic properties of traffic flows [7] demanded by Kerner and Rehborn [1]. More importantly, this model is able to describe the hysteretic phase transition to congested states with high traffic flow [8] (called ‘synchronized traffic’ [1]) which typically occurs behind on-ramps, gradients, or other bottlenecks of busy freeways [6].

According to the GKT model, the evolution of the vehicle density $\rho(x, t)$ as a function of time t and position x along the freeway is given by the continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial x} = \frac{Q_{\text{rmp}}}{nL}. \tag{1}$$

Here, $V(x, t)$ denotes the average velocity of the vehicles. At on-ramps (or off-ramps), the source term $Q_{\text{rmp}}/(nL)$ is given by the actually observed inflow $Q_{\text{rmp}} > 0$ from (or outflow $Q_{\text{rmp}} < 0$ to) the ramp, divided by the merging length L and by the number n of lanes. Otherwise it is zero, reflecting the conservation of the number of vehicles. The average velocity obeys the equation

$$\frac{\partial V}{\partial t} + \underbrace{V\frac{\partial V}{\partial x}}_{\text{transport term}} = \underbrace{-\frac{1}{\rho}\frac{\partial(\rho\theta)}{\partial x}}_{\text{pressure term}} + \underbrace{\frac{1}{\tau}(U - V)}_{\text{relaxation term}}. \tag{2}$$

According to this, the temporal change of the average velocity is given by a transport term (caused by a propagation of the velocity profile with V), a so-called pressure term (that reflects dispersion effects due to the finite velocity variance θ of the vehicles), and a relaxation term (describing the adaptation to a dynamic equilibrium velocity U with a certain relaxation time τ). In our GKT model, the analytically derived formula for the dynamical equilibrium velocity is

$$U = V_0\left[1 - \frac{\theta + \theta'}{2A(\rho_{\text{max}})}\left(\frac{\rho' T}{1 - \rho'/\rho_{\text{max}}}\right)^2 B(\delta_V)\right] \tag{3}$$

where V_0 is the desired (maximum) velocity, T the average time headway at large densities, and ρ_{max} the maximum vehicle density. A prime indicates that the corresponding variable is taken at the advanced ‘interaction point’ $x' = x + \gamma(1/\rho_{\text{max}} + TV)$ rather than at the actual position x. This accounts for the anisotropic anticipation behaviour of drivers. The monotonically increasing ‘Boltzmann factor’

$$B(\delta_V) = 2\left[\delta_V \frac{\mathrm{e}^{-\delta_V^2/2}}{\sqrt{2\pi}} + (1 + \delta_V^2)\int_{-\infty}^{\delta_V} \mathrm{d}y\, \frac{\mathrm{e}^{-y^2/2}}{\sqrt{2\pi}}\right] \tag{4}$$

can be derived from gas-kinetic formulae [7] and describes the dependence of the braking interaction on the dimensionless velocity difference $\delta_V = (V - V')/\sqrt{\theta + \theta'}$. Finally, the dynamics of the variance can be approximated by the constitutive relation

$$\theta(x, t) = \left[A_0 + \Delta A \tanh\left(\frac{\rho(x, t) - \rho_c}{\Delta\rho}\right)\right] V^2(x, t) \tag{5}$$

where the coefficients $A_0 = 0.008$, $\Delta A = 0.015$, $\rho_c = 0.28\rho_{max}$, and $\Delta\rho = 0.1\rho_{max}$ have been obtained from single-vehicle data [7].

The resulting velocity–density relation for this model in spatially homogeneous and stationary equilibrium reads

$$V_e(\rho) = \frac{\tilde{V}^2}{2V_0}\left(-1 + \sqrt{1 + \frac{4V_0^2}{\tilde{V}^2}}\right) \tag{6}$$

with

$$\tilde{V}(\rho) = \frac{1}{T}\left(\frac{1}{\rho} - \frac{1}{\rho_{max}}\right)\sqrt{\frac{A(\rho_{max})}{A(\rho)}}. \tag{7}$$

This also determines the equilibrium traffic flow by

$$Q_e(\rho) = \rho V_e(\rho) \tag{8}$$

which, for a given parameter set, is a one-dimensional curve. However, as will be shown in the following, the empirically observed two-dimensional region of 'synchronized' congested states can be reproduced by simulating a mixture of different vehicle types. Although it has not been stressed clearly enough, it is known from microsimulations that heterogeneous traffic produces considerable fluctuations of aggregate quantities like vehicle density and average velocity [9–11]. Nevertheless, we do not need to carry out microsimulations to account for the two-dimensional scattering of synchronized traffic states. It is sufficient to simulate traffic in a macroscopic way with empirically obtained boundary conditions, including the varying proportion of long vehicles ('trucks'). Thus, we do not need to assume other sources of fluctuations than the observed ones. A reasonable agreement with empirical data can already be reached by distinguishing only two vehicles types; short vehicles ('cars') and long ones ('trucks', with a length of at least 7 m). Each type is characterized by its own parameter set. For the cars we assume a desired velocity $V_0 = 112$ km h^{-1}, an average time headway $T = 1.0$ s at large densities, and a maximum density $\rho_{max} = 110$ vehicles km^{-1}. Trucks are described by the parameters $V_0 = 90$ km h^{-1}, $T = 5$ s, and $\rho_{max} = 100$ vehicles km^{-1}. The remaining model parameters are the same for both types: $\tau = 25$ s and $\gamma = 1.6$. The parameters in the constitutive relation (5) for the variance have also been chosen to be identical.

According to the philosophy of macroscopic models, we now define time-dependent 'effective' model parameters $X(t)$ as weighted averages of the respective car and truck parameters X_{car} and X_{truck}:

$$X(t) = p_{truck}(t)X_{truck} + [1 - p_{truck}(t)]X_{car}. \tag{9}$$

Here, $p_{truck}(t)$ is the proportion of trucks averaged over a time interval Δt around t (figure 2(*a*)). Although the approximations behind the resulting 'effective' macroscopic simulation model are rather crude, it yields a surprisingly good agreement with empirical data. Even better results are expected for macroscopic models which explicitly take into account different vehicle types and lane-changing interactions among the freeway lanes [12].

We simulated traffic flow on a section of the Dutch two-lane motorway A9 from Haarlem to Amsterdam (figure 1) from the detector cross section D1 (0 km) to D6 (5.7 km). For this purpose, the measured single-vehicle data were aggregated to one-minute averages of the velocity, traffic flow, and truck proportion (figure 2). Between 7:30 am (450 min) and 9:30 am (570 min) in the morning of November 2, 1994, we find transitions from a low-density regime to a high-density regime corresponding to transitions between free and congested traffic. Figure 3 illustrates that the congested state at D2 is connected with a considerable velocity drop, while the flow is decreased only by about 10%, both in the empirical data and in the

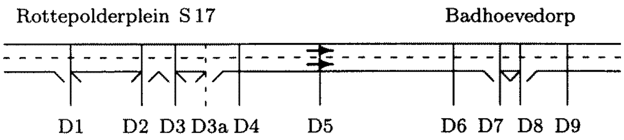

Figure 1. Overview of the evaluated stretch of the Dutch Highway A9 from Haarlem to Amsterdam.

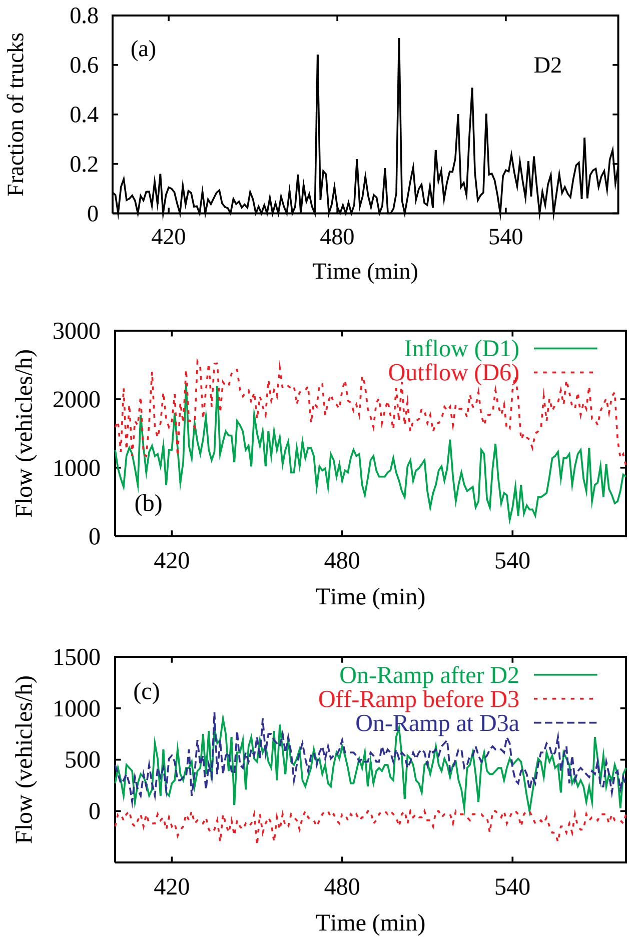

Figure 2. (*a*) Proportion of trucks, from one-minute averages. (*b*) Upstream and downstream boundary conditions for the flow, taken from measured one-minute data at the cross sections D1 and D6, respectively. (*c*) Flows of the three ramps in the considered section. The off-ramp at detector D1 was left out. It leads only to changes of the traffic situation upstream of the cross section D1 for which no data were available.

simulation. In addition, the congested traffic state relaxes to free traffic downstream of the on-ramps (figures 4(*c*) and (*d*)). A comparison with figures 1(*b*) and 3(*c*) of [8] suggests that the congestion in the investigated data corresponds to synchronized traffic.

As inflow and outflow boundary conditions, we used the data of the cross sections D1 and D6, respectively, as shown in figure 2(*b*). There are two on-ramps and one off-ramp in the considered section. For all ramps, we use empirical data of traffic flow $Q_{\rm rmp}$, divided by the

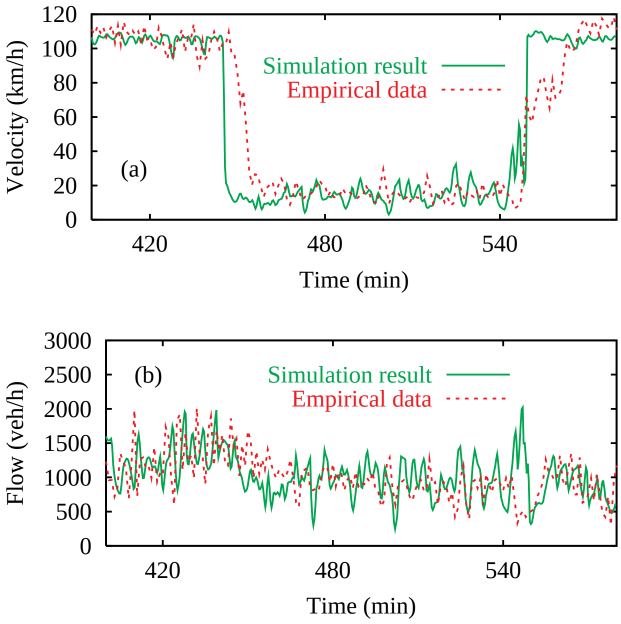

Figure 3. (*a*) Velocity, and (*b*) traffic flow at D2 according to the model, in comparison with the empirical one-minute data. The breakdown of velocity is a result of a dynamical transition, since neither the initial conditions, nor the boundary conditions, or the ramp flows used in the simulations contain any significant peaks.

number $n = 2$ of lanes [6] (figure 2(*c*)), and assume a merging length $L = 200$ m.

In the simulation, congested traffic first sets in at $t \approx 450$ min near the on-ramp at D3a. Some minutes later, the front of the congested state crosses the on-ramp at D2 (figure 3) which causes congested traffic upstream of it. All this agrees well with the observed data. We started the simulation 50 min earlier to eliminate any effects of initial conditions, and to show the spontaneous nature of the transition (figure 3(*a*)). In figure 4(*b*), free traffic flow before the breakdown ($t < 450$ min) and after the recovery ($t > 570$ min) is delineated by points at the low-density (left) side of the diagram, which more or less define a one-dimensional curve. In contrast, the congested traffic state is represented by points at the high-density (right) side, which are distributed over a two-dimensional region. In accordance with the mechanism of the formation of synchronized traffic proposed in [6], the congested state upstream of D2 has a lower flow and a higher density (figure 4(*a*)) than that between D2 and D3a. The congested states are sustained for nearly two hours, until the inflows from both the main road and the on-ramps are considerably decreased, which shows the hysteretic nature of the transition.

Summarizing our results, one can say that the macroscopic GKT model allows us to simulate synchronized traffic, including the associated scattering of the flow–density data in the congested regime. Simulations of this model with only one vehicle type [6, 13] suggested that the phenomenon of synchronized traffic as such (i.e., high traffic flows at low velocities) does not depend on the existence of different types of vehicles. However, as is often the case for self-organized non-chaotic patterns resulting from deterministic dynamics, the flow–density diagram is essentially one-dimensional.

In this letter, we showed that a realistic scattering in the flow–density plane can be simulated by distinguishing several vehicle types with different parameter sets, the measured

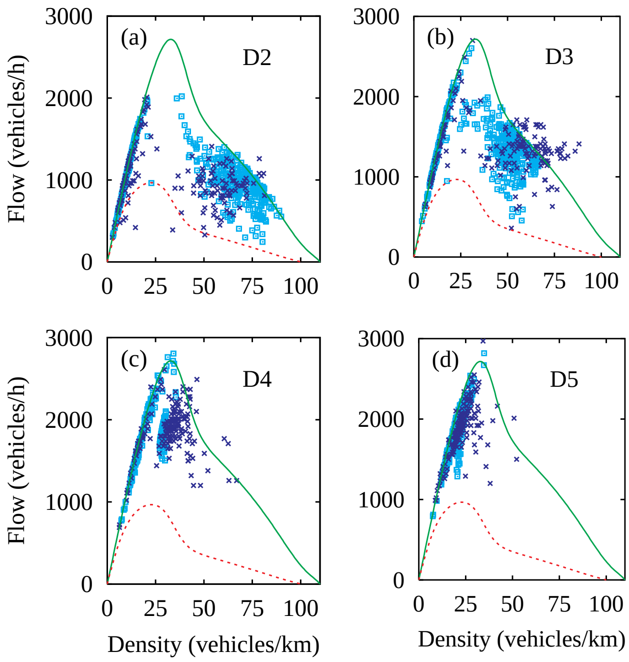

Figure 4. The displayed points in density–flow space correspond to empirical one-minute data (dark crosses) and related simulation results (grey boxes), separately for the cross sections D2–D5. The simulations manage to reproduce both the quasi-linear flow–density relation at small densities and the scattering over a two-dimensional region at high densities. For comparison, we have displayed the equilibrium flow–density relations for traffic consisting of 100% cars (——), and 100% trucks (– – –).

proportions of which are the weights for determining the time-dependent 'effective' parameter set. A reasonable agreement with empirical data from Dutch highways is already obtained for two different vehicle types, cars and trucks. Our results also indicate that, when studying dynamical phenomena in empirical traffic data, it is highly recommended to thoroughly analyse the proportion of trucks, which shows surprisingly large variations (figure 2(*a*)).

Notice that the assumed parameter variations due to a changing truck fraction can explain both the relatively low scattering of flow–density data in the low-density regime and wide scattering in the regime of congested traffic. While a small amount of scattering for free traffic at low densities is caused mainly by variations of the individual desired velocity (with a standard deviation of about 10% of the mean value), the main reason for the considerable scattering for congested traffic at densities above 30 vehicles km^{-1} are the variations of the time headway (which are of the order of 100%). There are other effects that influence scattering of congested traffic (e.g., lane changes), but they will only increase the scattering of the flow–density data.

Finally, we mention that the equations of the GKT traffic model together with relation (9) for the stochastic quantities V_e, T, and ρ_{max}, represent stochastic partial equations with multiplicative noise. They may serve as prototype for introducing stochasticity into macroscopic equations in a controlled and empirically justified manner.

The authors wish to thank the BMBF (research project SANDY, grant no 13N7092) and the DFG (Heisenberg scholarship He 2789/1-1) for financial support. They are also grateful to

Henk Taale and the Dutch Ministry of Transport, Public Works and Water Management for supplying the freeway data.

References

[1] Kerner B S and Rehborn H 1996 *Phys. Rev.* E **53** R4275
[2] Kerner B S 1998 *Proc. 3rd Int. Symp. Highway Capacity* vol II, ed R Rysgaard (Copenhagen: Road Directorate)
[3] Krauß S 1998 Microscopic modelling of traffic flow *PhD Thesis* (Cologne: DLR) FB 98-08
[4] Lenz H, Wagner C K and Sollacher R 1998 *Eur. J. Phys.* at press
[5] Schreckenberg M 1998 to be published
[6] Helbing D and Treiber M 1998 *Phys. Rev. Lett.* **81** 3042
[7] Treiber M, Hennecke A and Helbing D 1999 *Phys. Rev.* E **59** at press
[8] Kerner B S and Rehborn H 1997 *Phys. Rev. Lett.* **49** 4030
[9] Nagel K, Wolf D E, Wagner P and Simon P 1998 *Phys. Rev.* E **58** 1425
[10] Helbing D and Schreckenberg M 1998 Cellular automata simulating experimental properties of traffic flows *Phys. Rev.* submitted
[11] Helbing D and Huberman B A 1998 Moving like a solid block *Nature* at press
[12] Shvetsov V and Helbing D 1998 Macroscopic dynamics of multi-lane traffic *Phys. Rev.* E submitted
[13] Helbing D and Treiber M 1999 Numerical simulation of macroscopic traffic equations *Comput. Sci. Eng.* at press